Rationalise the denominator

When dealing with surds and fractions, it is commonly accepted that we do not like surds as a denominator. Because of this me need to rationalise the denominator. That means we make the denominator a rational number.

Example 1

Rationalise the denominator of

$$\frac{3}{\sqrt{2}}$$

We have $\sqrt{2}$ as the denominator. If we multiply the fraction by $\frac{\sqrt{2}}{\sqrt{2}}$ (which equals 1) we can eliminate the surd as a denominator without changing the value of the fraction.

$$\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{4}} = \frac{3\sqrt{2}}{2}$$

Now obviously we usually get more complex denominators to rationalise. Let us look at a more complex one and see where we go with this.

Example 2

Rationalise the denominator of

$$\frac{5}{2+\sqrt{3}}$$

As we have an integer and a surd as the denominator we need to ensure that whatever we multiply the denominator by, we eliminate any surds. Let us solve this by thinking back to when we multiplied out factors of quadratics

$(x+y)(x-y)$ becomes $x^2 + xy - xy - y^2$ and simplifies to $x^2 - y^2$

This would mean, for our example, we would need to multiply by $\left(2-\sqrt{3}\right)$.

$$\frac{5}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{5(2-\sqrt{3})}{2^2-\sqrt{3}^2} = \frac{10-5\sqrt{3}}{4-3} = \frac{10-5\sqrt{3}}{1} = 10-5\sqrt{3}$$

Rationalise the denominator

Example 3

Rationalise the denominator of

$$\frac{6\sqrt{2}}{\sqrt{2}-\sqrt{6}}$$

Now as we have the sign of the denominator negative, we simple use $\left(\sqrt{2}+\sqrt{6}\right)$

$$\frac{6\sqrt{2}}{\sqrt{2}-\sqrt{6}} \times \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}} = \frac{6\sqrt{2}(\sqrt{2}+\sqrt{6})}{\sqrt{2}^2-\sqrt{6}^2} = \frac{6(2+\sqrt{2}\sqrt{6})}{2-6} = \frac{12+6\sqrt{12}}{-4} = \frac{12+12\sqrt{2}}{-4} = -3-3\sqrt{2}$$

Exercise 1

1) $\frac{2}{\sqrt{3}}$

2) $\frac{4}{\sqrt{2}}$

3) $\frac{\sqrt{6}}{\sqrt{3}}$

4) $\frac{\sqrt{6}}{3\sqrt{2}}$

5) $\frac{3}{3+\sqrt{3}}$

6) $\frac{5}{2+\sqrt{6}}$

7) $\frac{7}{2-\sqrt{5}}$

8) $\frac{5}{\sqrt{5}-\sqrt{7}}$

9) $\frac{\sqrt{7}}{\sqrt{5}-\sqrt{10}}$

10) $\frac{\sqrt{5}}{\sqrt{2}+\sqrt{3}}$

11) $\frac{3+\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

12) $\frac{5-\sqrt{3}}{\sqrt{6}+\sqrt{2}}$

13) $\frac{7+\sqrt{5}}{\sqrt{5}-\sqrt{2}}$

14) $\frac{5-\sqrt{6}}{\sqrt{8}-\sqrt{3}}$

15) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

16) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{10}-\sqrt{6}}$

17) $\frac{\sqrt{6}-\sqrt{7}}{\sqrt{6}+\sqrt{3}}$

18) $\frac{3\sqrt{3}-2\sqrt{2}}{\sqrt{13}-\sqrt{7}}$

19) $\frac{3\sqrt{6}-\sqrt{2}}{6\sqrt{3}-\sqrt{2}}$

20) $\frac{\sqrt{10}-5\sqrt{2}}{\sqrt{10}+2\sqrt{5}}$